Heat and Mass Transfer at a General Three-Dimensional Stagnation Point

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Theme

POOTS[1] presented detailed results for compressible laminar three-dimensional stagnation-point flow of a gas with constant properties (i.e., density-viscosity product $\rho\mu$ is constant and Prandtl number σ is unity). This work was extended to include mass transfer (injection only) and nonunity Prandtl number by Libby.[2] While Poots[1] solved the governing equations numerically, using the forward integration scheme, Libby[2] used the quasilinearization technique. In recent years, Libby and his coworkers[3,4] have extended the quasilinearization technique to study the laminar boundary-layer problem with variable gas properties at an axisymmetric stagnation point with hydrogen injection.

This three-dimensional problem with variable gas properties, employing model gas equations ($\mu \propto h^{\omega}$, $\rho \propto h^{-1}$, $\sigma = 0.7$, where h is the enthalpy and ω is the viscosity exponent) without mass transfer, was considered by Wortman[5,6] and Wortman et al.[7] With mass transfer (i.e., only injection with or without a foreign gas), it was considered by Wortman and Mills[8] and Wortman.[9] The governing equations in Refs. 5–9 were solved by a method based on functional analysis.

The main objective here is to obtain values of critical wall parameters which were not obtained by the previous studies of this problem. With this in mind, the solution of the preceding problem with variable gas properties for both suction and injection has been obtained by the use of the method of parametric differentiation.[10–13] The values of the shear stress and heat transfer parameters for various values of physical parameters of the flow are presented in tabular form. These results show that, irrespective of the nature of the stagnation point, the critical wall values are affected appreciably with the variations of ω, the exponent in power law, only at low wall temperatures, their variations with ω being considerably small at high values of the wall temperature. Another important feature that is observed is that the variation of the density-viscosity product (i.e., $\omega \neq 1$) across the boundary layer gives rise to a point of inflexion in the velocity and enthalpy profiles.

Contents

The basic differential equations describing compressible variable-properties ($\mu \propto h^{\omega}$, $\rho \propto h^{-1}$, $\sigma = 0.7$) three-dimensional stagnation-point boundary-layer flows have been discussed thoroughly by Libby,[2] so that in the final form they can be expressed as

$$f''' + (\omega-1)g'f''/g + [(f+c\phi)f'' + g - f'^2]g^{1-\omega} = 0 \quad (1a)$$

$$\phi''' + (\omega-1)g'\phi''/g + [(f+c\phi)\phi'' + c(g-\phi'^2)]g^{1-\omega} = 0 \quad (1b)$$

$$g'' + (\omega-1)g'^2/g + \sigma[f+c\phi]g'g^{1-\omega} = 0 \quad (1c)$$

with the following boundary conditions

$$f(0) = f_w,\ f'(0) = \phi(0) = \phi'(0) = 0,\ g(0) = g_w$$
$$f'(\infty) = \phi'(\infty) = g(\infty) = 1 \quad (2)$$

Here $c = b/a$, where a and b are the velocity gradients at the stagnation point in the x and y directions, respectively; f, ϕ, and g are the dependent similarity variables given by

$$f' = u/ax, \quad \phi' = v/by, \quad g = h/h_e \quad (3)$$

primes denote differentiation with respect to the independent similarity variable

$$\eta = (\rho_e a/\mu_e)^{1/2} \int_0^z (\rho/\rho_e)\, dz$$

while suffixes w and e refer to conditions at the wall and the edge of the boundary layer, respectively. It may be noted that $\omega = 1.0$ yields the familiar constant density-viscosity product simplification; $\omega = 0.7$ is appropriate for low-temperature flows while $\omega = 0.5$ may be considered as a limiting value for high temperature flows.[14]

The nonlinear two-point boundary-value problem posed by Eqs. (1) and (2) is solved through an application of the method of parametric differentiation[10–13] for a range of values of c, f_w, and g_w, the parameters characterizing the nature of the stagnation point, mass transfer, and wall temperature, respectively, for $\sigma = 0.7$ and $\omega = 0.5, 0.7, 1.0$. The detailed description of the method of parametric differentiation along with its applications to various problems is given in Refs. 10–13.

Tables 1 and 2 show the effect of the variations of c, f_w, g_w, and ω on $f''(0)$, $\phi''(0)$, and $g'(0)$, the shear stress and heat

Table 1 Effect[a] of mass transfer ($\omega = 1$)

c	f_w	$f''(0)$ $g_w = 0$	$f''(0)$ $g_w = 1$	$\phi''(0)$ $g_w = 0$	$\phi''(0)$ $g_w = 1$	$g'(0)$ $g_w = 0$	$g'(0)$ $g_w = 1$
	−0.5	0.3050	0.9693	0.2043	0.2957	0.2194	0.0
		(0.2966)	(0.9692)	(0.1984)	(0.2950)	(0.2102)	
0	0.0	0.6071	1.2326	0.4970	0.5705	0.4362	0.0
		(0.6071)	(1.2326)	(0.4970)	(0.5705)	(0.4362)	
	0.5	0.9702	1.5417	0.8705	0.9215	0.6966	0.0
	1.0	1.3726	1.8893	1.2888	1.3232	0.9827	0.0
	−0.5	0.4616	1.0340	0.4616	1.0340	0.3679	0.0
		(0.4614)	(1.0340)	(0.4614)	(1.0340)	(0.3677)	
1	0.0	0.7821	1.3119	0.7821	1.3119	0.6058	0.0
		(0.7821)	(1.3119)	(0.7821)	(1.3119)	(0.6058)	
	0.5	1.1462	1.6309	1.1462	1.6309	0.8692	0.0
	1.0	1.5419	1.9837	1.5419	1.9837	1.1509	0.0
	−0.5	0.3662	1.0095	−0.2221	−0.7648	0.2733	0.0
		(0.3774)	(1.0110)	(−0.2323)	(−0.7800)	(0.2836)	
−1	0.0	0.6631	1.2717	−0.2890	−0.7971	0.4863	0.0
		(0.6631)	(1.2717)	(−0.2890)	(−0.7971)	(0.4863)	
	0.5	1.0088	1.5730	−0.3176	−0.7517	0.7303	0.0
	1.0	1.3913	1.9081	−0.2717	−0.6105	0.9975	0.0

[a] The values in the parentheses are the values obtained by Libby.[2]

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Table 2 Effect of mass transfer ($g_w = 0.2$)

c	f_w	$f''(0)$ $\omega = 0.5$	$f''(0)$ $\omega = 0.7$	$f''(0)$ $\omega = 1.0$	$\phi''(0)$ $\omega = 0.5$	$\phi''(0)$ $\omega = 0.7$	$\phi''(0)$ $\omega = 1.0$	$g'(0)$ $\omega = 0.5$	$g'(0)$ $\omega = 0.7$	$g'(0)$ $\omega = 1.0$
0	−0.5	0.2950	0.3522	0.4579	0.1487	0.1769	0.2289	0.1203	0.1447	0.1927
	0.0	0.4144	0.5221	0.7453	0.2731	0.3509	0.5156	0.1906	0.2451	0.3625
	0.5	0.5594	0.7282	1.0933	0.4309	0.5723	0.8829	0.2774	0.3676	0.5680
	1.0	0.7235	0.9603	1.4827	0.6099	0.8228	1.2974	0.3751	0.5046	0.7957
1	−0.5	0.3568	0.4359	0.5893	0.3568	0.4359	0.5893	0.1777	0.2220	0.3109
	0.0	0.4871	0.6194	0.8956	0.4871	0.6194	0.8956	0.2569	0.3331	0.4957
	0.5	0.6362	0.8297	1.2474	0.6362	0.8297	1.2474	0.3456	0.4576	0.7029
	1.0	0.8002	1.0605	1.6327	0.8002	1.0605	1.6327	0.4415	0.5920	0.9259
−1	−0.5	0.3143	0.3822	0.5124	−0.1696	−0.2206	−0.3182	0.1354	0.1693	0.2378
	0.0	0.4300	0.5469	0.7905	−0.1731	−0.2353	−0.3657	0.2018	0.2642	0.3983
	0.5	0.5665	0.7408	1.1185	−0.1453	−0.2055	−0.3380	0.2803	0.3755	0.5856
	1.0	0.7202	0.9587	1.4851	−0.0788	−0.1197	−0.2145	0.3684	0.4997	0.7933

transfer parameters. It is evident from Table 1 that there is good agreement between the results of the present investigation and those of Ref. 2, except in the case of injection when $g_w = 0$ (for $f_w = -0.5$, the maximum difference is about 4.5%). It has been found that the effect of changing ω is appreciable only for small g_w, no matter what the value of c is. (These results are available in Table 2 of the backup paper). Table 2 presents the mass transfer effectiveness on both the shear stress and heat transfer parameters, while Ref. 8 gives the effect of mass injection on heat transfer in graphical form. It has also been found that the velocity and enthalpy profiles for $\omega = 0.5$ and 0.7 have a point of inflexion, which is not present in the $\omega = 1$ profiles as evidenced by a maximum in $f''(\eta)$, $\phi''(\eta)$, $g'(\eta)$, whether there is mass transfer or not. It is important to note that similar effects have been observed by Gross and Dewey[14] on using the power law relation for two-dimensional and axisymmetric stagnation-point flows. Therefore, it can be concluded that the linear viscosity-temperature relation does not hold good at low wall temperature.

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